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Effects of thermal radiation and variable fluid viscosity on free convective flow and heat transfer past a porous stretching surface

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Abstract

Free convective boundary layer flow and heat transfer of a fluid with variable viscosity over a porous stretching vertical surface in presence of thermal radiation is considered. Fluid viscosity is assumed to vary as a linear function of temperature. The symmetry groups admitted by the corresponding boundary value problem are obtained by using a special form of Lie group transformations viz. scaling group of transformations. A third-order and a second-order coupled ordinary differential equations corresponding to the momentum and the energy equations are obtained. These equations are then solved numerically. It is found that the skin-friction decreases and heat transfer rate increases due to the suction parameter. Opposite nature is noticed in case of blowing. With the increase of temperature-dependent fluid viscosity parameter (i.e. with decreasing viscosity), the fluid velocity increases but the temperature decreases at a particular point of the sheet. Due to suction (injection) fluid velocity decreases (increases) at a particular point of the surface. Effects of increasing Prandtl number as well as radiation parameter on the velocity boundary layer is to suppress the velocity field and the temperature decreases with increasing value of Prandtl number. Due to increase in thermal radiation parameter, temperature at a point of the surface is found to decrease.

Keywords: Scaling group of transformations; Free convective flow; Porous stretching surface; Temperature-dependent fluid viscosity; Suction/blowing; Thermal radiation

1. Introduction

The study of hydrodynamic flow and heat transfer over a porous stretching sheet has gained considerable attention due to its vast applications in the industry and important bearings on several technological and natural processes. The production of sheeting material arises in a number of industrial manufacturing processes and includes both metal and polymer sheets. It is well known that the flow in a boundary layer separates in the regions of adverse pressure gradient and the occurrence of separation has several undesirable effects in so far as it leads to increase in the drag on the body immersed in the flow and adversely affects the heat transfer from the surface of the body. Several methods have been developed for the purpose of artificial control of flow separation. Separation can be prevented by suction as the low-energy fluid in the boundary layer is removed [1,2]. On the contrary, the wall shear stress and hence the friction drag is reduced by blowing.

Free convective phenomenon has been the object of extensive research. The importance of this phenomenon is increasing day by day due to the enhanced concern in science and technology about buoyancy induced motions in the atmosphere, the bodies in water and quasisolid bodies such as earth. Natural convection flows driven by temperature differences are very much interesting in case of Industrial applications. Buoyancy plays an important role where the temperature differences between land and air give rise

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Nomenclature

A	fluid viscosity variation parameter
F	non-dimensional stream function
F^*	variable
F'	first-order derivative with respect to η
F''	second-order derivative with respect to η
F'''	third-order derivative with respect to η
k^*	absorption coefficient
Pr	Prandtl number
p, q	variables
q_r	radiative heat flux
T	temperature of the fluid
T_w	temperature of the wall of the surface
T_∞	free-stream temperature
u, v	components of velocity in x and y directions
z	variable
β	volumetric coefficient of thermal expansion
η	similarity variable
Γ	Lie-group transformations
κ	the coefficient of thermal diffusivity
μ	dynamic viscosity
μ^*	reference viscosity
v^*	reference kinematic viscosity
ψ	stream function
ψ^*	variable
σ	Stefan–Boltzman constant
ρ	density of the fluid
θ	non-dimensional temperature
$\theta^*, \bar{\theta}$	variables
θ'	first-order derivative with respect to η
θ''	second-order derivative with respect to η

Greek symbols

α_1, α_2, α_3, α_4, α_5, α_6, α', α'' transformation parameters

β', β'' transformation parameters

to a complicated flow and in enclosures such as ventilated and heated rooms (Elbashbeshy and Bazid [3]).

So such type of problem, that we are dealing with, is very much useful to polymer technology and metallurgy. Cheng and Minkowycz [4] and Cheng [5] studied the free convective flow in a saturated porous medium. Wilks [6] had studied the combined forced and free convection flow along a semi-infinite plate extending vertically upwards with its leading edge horizontal. Boutros et al. [7] solved the steady free convective boundary layer flow on a non-isothermal vertical plate. Recently, any studies were made on the steady free convective boundary layer flow on moving vertical plates considering the effect of buoyancy forces on the boundary layer Chen and Strobel [8], Ramachandran et al. [9], Lee and Tsai [10].

The radiative effects have important applications in physics and engineering particularly in space technology and high temperature processes. But very little is known about the effects of radiation on the boundary layer. Thermal radiation effects may play an important role in controlling heat transfer in polymer processing industry where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, power generation systems are some important applications of radiative heat transfer from a vertical wall to conductive gray fluids. The effect of radiation on heat transfer problems have studied by Hossain and Takhar [11], Takhar et al. [12], Hossain et al. [13].

In all of the above mentioned studies, fluid viscosity was assumed to be constant. However, it is known that the physical properties of fluid may change significantly with temperature. For lubricating fluids, heat generated by the internal friction and the corresponding rise in temperature affects the viscosity of the fluid and so the fluid viscosity can no longer be assumed constant. The increase of temperature leads to a local increase in the transport phenomena by reducing the viscosity across the momentum boundary layer and so the heat transfer rate at the wall is also affected. Therefore, to predict the flow behaviour accurately it is necessary to take into account the viscosity variation for incompressible fluids. Gary et al. [14] and Mehta and Sood [15] showed that, when this effect is included the flow characteristics may changed substantially compared to the constant viscosity assumption. Recently Mukhopadhyay et al. [16] investigated the MHD boundary layer flow with variable fluid viscosity over a heated stretching sheet.

The present work deals with free convective flow and radiative heat transfer of viscous incompressible fluid having variable viscosity over a stretching porous vertical plate. The system remains invariant due to some relations among the parameters of the scaling group of transformations. Using these invariants, a third-order and a second-order coupled ordinary differential equations corresponding to the momentum and the energy equations are derived. These equations are solved numerically using shooting method. The effects of the temperature-dependent fluid viscosity parameter, suction/injection parameter, the influence of Prandtl number and radiation parameter on velocity and temperature fields of the fluid are investigated and analysed with the help of their graphical representations.

2. Equations of motion

We consider a free convective, laminar boundary layer flow and heat transfer of viscous incompressible fluid over a porous stretching sheet emerging out of a slit at origin

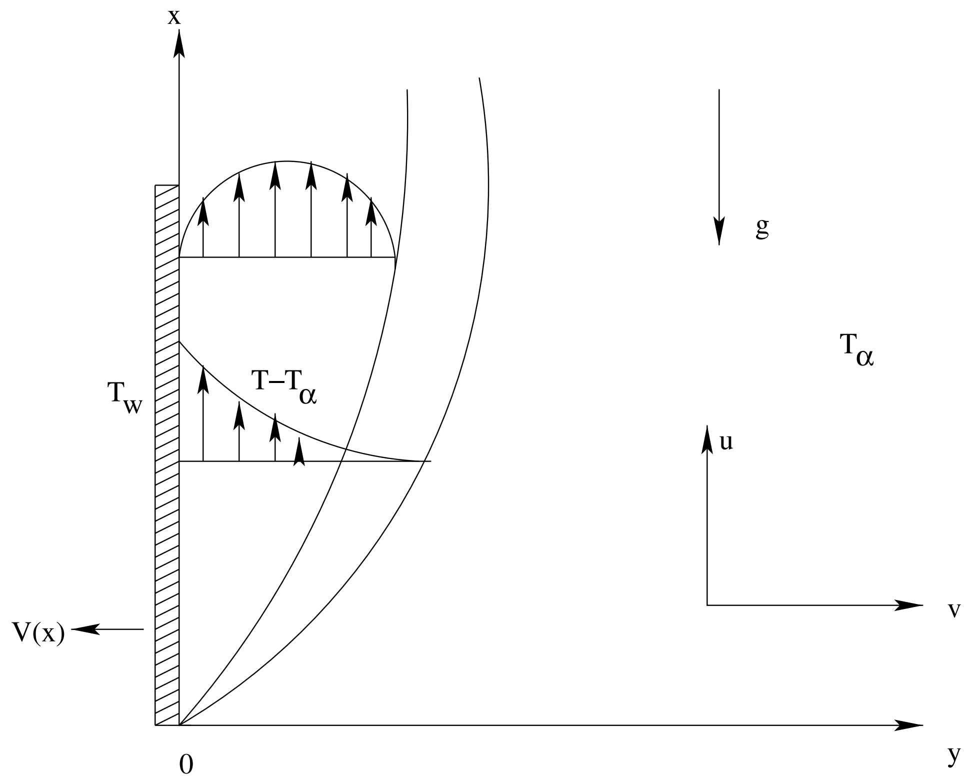

Fig. 1. Physical model of boundary layer flow over a vertical stretching surface.

$(x = 0, y = 0)$ and moving with non-uniform velocity $U(x)$ in presence of thermal radiation (Fig. 1).

The governing equations of such type of flow are, in the usual notations,

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{3}$$

when the viscous dissipation term in the energy equation is neglected (as the fluid velocity is very low). Here u and υ are the components of velocity respectively in the x and y directions, μ is the coefficient of fluid viscosity, ρ is the fluid density (assumed constant), T is the temperature, κ is the thermal conductivity of the fluid, β is the volumetric coefficient of thermal expansion, g is the gravity field, T_∞ is the temperature at infinity.

Using Rosseland approximation for radiation (Brewster [17]) we can write $q_r = -\frac{4\sigma_1}{3k^*}\frac{\partial T^4}{\partial y}$ where σ_1 is the Stefan–Boltzman constant, k^* is the absorption coefficient.

Assuming that the temperature difference within the flow is such that T^4 may be expanded in a Taylor series and expanding T^4 about T_∞ and neglecting higher orders we get $T^4 \equiv 4T_\infty^3 T - 3T_\infty^4$. Therefore, the Eq. (3) becomes

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma_1 T_\infty^3}{3\rho c_p k^*}\frac{\partial^2 T}{\partial y^2}. \tag{4}$$

2.1. Boundary conditions

The appropriate boundary conditions for the problem are given by

$$u = U(x), \upsilon = -V(x), \quad T = T_w \text{ at } y = 0, \tag{5}$$

$$u \to 0, T \to T_\infty \text{ as } y \to \infty. \tag{6}$$

where $U(x)$ is the stream wise velocity and $V(x)$ is the velocity of suction of the fluid, T_w is the wall temperature.

2.2. Method of solution

We now introduce the following relations for u, υ and θ as

$$u=\frac{\partial \psi}{\partial y}, \quad \upsilon=-\frac{\partial \psi}{\partial x} \text{ and } \theta=\frac{T-T_\infty}{T_w-T_\infty} \tag{7}$$

where φ is the stream function.

The stream wise velocity and the suction/injection velocity are taken as

$$U(x) = cx^m, \quad V(x) = V_0 x^{\frac{m-1}{2}}.$$

Here $c(>0)$ is constant, T_w is the wall temperature, the power-law exponent m is also constant. In this study we take $c = 1$.

The temperature-dependent fluid viscosity is given by (Batchelor [18]),

$$\mu = \mu^*[a + b(T_w - T)] \tag{8}$$

where μ^* is the constant value of the coefficient of viscosity far away from the sheet and *a, b* are constants and $b(>0)$.

For a viscous fluid, Ling and Dybbs [19] suggest a viscosity dependence on temperature T of the form $\mu = \mu_\infty/[1 + \gamma\,(T - T_\infty)]$ where γ is a thermal property of the fluid and μ_∞ is the viscosity away from the hot sheet. This relation does not differ at all with our formulation.

The range of temperature, i.e. $(T_w - T_\infty)$ studied here is (0–23 °C).

Using the relations (6) in the boundary layer Eq. (2) and in the energy Eq. (3) we get the following equations

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = -A\nu^*\frac{\partial\theta}{\partial y}\frac{\partial^2\psi}{\partial y^2} + \nu^*[a + A(1-\theta)]\frac{\partial^3\psi}{\partial y^3} + g\beta\frac{A}{b}\theta, \tag{9}$$

and

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y} = (\frac{\kappa}{\rho c_p} + \frac{16\sigma T_\infty^3}{3\rho c_p k^*})\frac{\partial^2\theta}{\partial y^2} \tag{10}$$

where $A = b(T_w - T_\infty)$, $\nu^* = \frac{\mu^*}{\rho}$.

The boundary conditions equations (5) and (6) then become

$$\frac{\partial\psi}{\partial y} = x^m, \quad \frac{\partial\psi}{\partial x} = V_0 x^{\frac{m-1}{2}}, \quad \theta = 1 \text{ at } y = 0. \tag{11}$$

$$\frac{\partial\psi}{\partial y} \to 0, \theta \to 0 \text{ as } y \to \infty. \tag{12}$$

2.3. Scaling group of transformations

We now introduce the simplified form of Lie-group transformations namely, the scaling group of transformations (Mukhopadhyay et al. [16]),

$$\Gamma : x^* = xe^{\varepsilon\alpha_1}, \quad y^* = ye^{\varepsilon\alpha_2}, \quad \psi^* = \psi e^{\varepsilon\alpha_3},$$
$$u^* = ue^{\varepsilon\alpha_4}, \quad \upsilon^* = \upsilon e^{\varepsilon\alpha_5}, \quad \theta^* = \theta e^{\varepsilon\alpha_6}. \tag{13}$$

Eq. (13) may be considered as a point-transformation which transforms co-ordinates $(x, y, \psi, u, \upsilon, \theta)$ to the co-ordinates $(x^*, y^*, \psi^*, u^*, \upsilon^*, \theta^*)$.

Substituting (13) in (9) and (10) we get,

$$e^{\varepsilon(\alpha_1+2\alpha_2-2\alpha_3)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial x^*\partial y^*} - \frac{\partial\psi^*}{\partial x^*}\frac{\partial^2\psi}{\partial y^{*2}}\right) = -A\nu^* e^{\varepsilon(3\alpha_2-\alpha_3-\alpha_6)}\frac{\partial\theta^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial y^{*2}} + (a+A)\nu^* e^{\varepsilon(3\alpha_2-\alpha_3)}\frac{\partial^3\psi^*}{\partial y^{*3}} - \nu^* A e^{\varepsilon(3\alpha_2-\alpha_3-\alpha_6)}\theta^*\frac{\partial^3\psi^*}{\partial y^{*3}} + g\beta\frac{A}{b}e^{-\varepsilon\alpha_6}\theta^*, \tag{14}$$

$$e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_6)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial\theta^*}{\partial x^*} - \frac{\partial\psi^*}{\partial x^*}\frac{\partial\theta^*}{\partial y^*}\right) = \left(\frac{\kappa}{\rho c_p} + \frac{16\sigma T_\infty^3}{3\rho c_p k^*}\right)e^{\varepsilon(2\alpha_2-\alpha_6)}\frac{\partial^2\theta^*}{\partial y^{*2}}. \tag{15}$$

The system will remain invariant under the group of transformations Γ, we would have the following relations among the parameters, namely

$$\alpha_1 + 2\alpha_2 - 2\alpha_3 = 3\alpha_2 - \alpha_3 - \alpha_6 = 3\alpha_2 - \alpha_3 = -\alpha_6$$
$$\text{and } \alpha_1 + \alpha_2 - \alpha_3 - \alpha_6 = 2\alpha_2 - \alpha_6.$$

These relations give $\alpha_6 = 0$, $\alpha_2 = \frac{1}{4}\alpha_1 = \frac{1}{3}\alpha_3$.

The boundary conditions yield $\alpha_4 = m\alpha_1 = \frac{1}{2}\alpha_1$, $\alpha_5 = \frac{m-1}{2}\alpha_1 = -\frac{1}{4}\alpha_1$ (as $m = \frac{1}{2}$).

In view of these, the boundary conditions become

$$\frac{\partial\psi^*}{\partial y^*} = x^{*\frac{1}{2}}, \quad \frac{\partial\psi^*}{\partial x^*} = V_0 x^{*(-\frac{1}{4})}, \quad \theta^* = 1 \text{ at } y^* = 0 \tag{16}$$

and

$$\frac{\partial\psi^*}{\partial y^*} \to 0, \theta^* \to 0 \text{ as } y^* \to \infty. \tag{17}$$

The set of transformations Γ reduces to

$$x^* = xe^{\varepsilon\alpha_1}, \quad y^* = ye^{\varepsilon\frac{\alpha_1}{4}}, \quad \psi^* = \psi e^{\varepsilon\frac{3\alpha_1}{4}}, \quad u^* = ue^{\varepsilon\frac{\alpha_1}{2}},$$
$$\upsilon^* = \upsilon e^{-\varepsilon\frac{\alpha_1}{4}}, \quad \theta^* = \theta.$$

Expanding by Taylor's method in powers of ε and keeping terms up to the order ε we get

$$x^* - x = x\varepsilon\alpha_1, \quad y^* - y = y\varepsilon\frac{\alpha_1}{4},$$
$$\psi^* - \psi = \psi\varepsilon\frac{3\alpha_1}{4}, \quad u^* - u = u\varepsilon\frac{\alpha_1}{2},$$
$$\upsilon^* - \upsilon = -\upsilon\varepsilon\frac{\alpha_1}{4}, \quad \theta^* - \theta = 0.$$

In terms of differentials these yield

$$\frac{dx}{\alpha_1 x} = \frac{dy}{\frac{\alpha_1}{4}y} = \frac{d\psi}{\frac{3\alpha_1}{4}\psi} = \frac{du}{\frac{\alpha_1}{2}u} = \frac{d\upsilon}{-\frac{\alpha_1}{4}\upsilon} = \frac{d\theta}{0}.$$

Solving the above equations we get,

$$y^* x^{*\frac{-1}{4}} = \eta, \quad \psi^* = x^{*\frac{3}{4}}F(\eta), \quad \theta^* = \theta(\eta). \tag{18}$$

With the help of these relations, the (14) and (15) become

$$2F'^2 - 3FF'' = -4A\nu^*\theta' F'' + 4(a+A)\nu^* F''' - 4\nu^* A\theta F''' + 4\frac{g\beta}{b}A\theta \tag{19}$$

and

$$4\left(\frac{\kappa}{\rho c_p} + \frac{16\sigma T_\infty^3}{3\rho c_p k^*}\right)\theta'' + 3F\theta' = 0. \tag{20}$$

The boundary conditions take the following form

$$F' = 1, \quad F = \frac{4V_0}{3}, \quad \theta = 1 \text{ at } \eta = 0 \tag{21}$$

and

$$F' \to 0, \theta \to 0 \text{ as } \eta \to \infty. \quad (22)$$

Again, we introduce the following transformations for η, F and θ in Eqs. (19) and (20):

$$\eta = (g\beta/b)^{a_1} v^{*b_1} \eta^*, \quad F = (g\beta/b)^{a'_1} v^{*b'_1} F^*,$$
$$\theta = (g\beta/b)^{a''_1} v^{*b''_1} \bar{\theta}. \quad (23)$$

Taking $F^* = f$ and $\bar{\theta} = \theta$ the Eqs. (19) and (20) finally take the following form:

$$4(a+A)f''' - 4A\theta f''' - 4A\theta' f'' + 4A\theta + 3ff'' - 2f'^2 = 0, \quad (24)$$

and

$$\frac{4}{Pr}\left(1 + \frac{4}{3N}\right)\theta'' + 3f\theta' = 0 \quad (25)$$

where $Pr = \frac{v^* \rho c_p}{\kappa} = \frac{\mu^* c_p}{\kappa}$ is the Prandtl number, $N = \frac{\kappa k^*}{4\sigma T_\infty^3}$ is the radiation parameter. The boundary conditions take the following forms

$$f' = 1, \quad f = S, \quad \theta = 1 \text{ at } \eta^* = 0 \quad (26)$$

and

$$f' \to 0, \theta \to 0 \text{ as } \eta^* \to \infty \quad (27)$$

where $S = \frac{4}{3} V_0 (\frac{g\beta}{b})^{\frac{-1}{4}} v^{\frac{-1}{2}}$, $S > 0$ corresponds to suction and $S < 0$ corresponds to blowing.

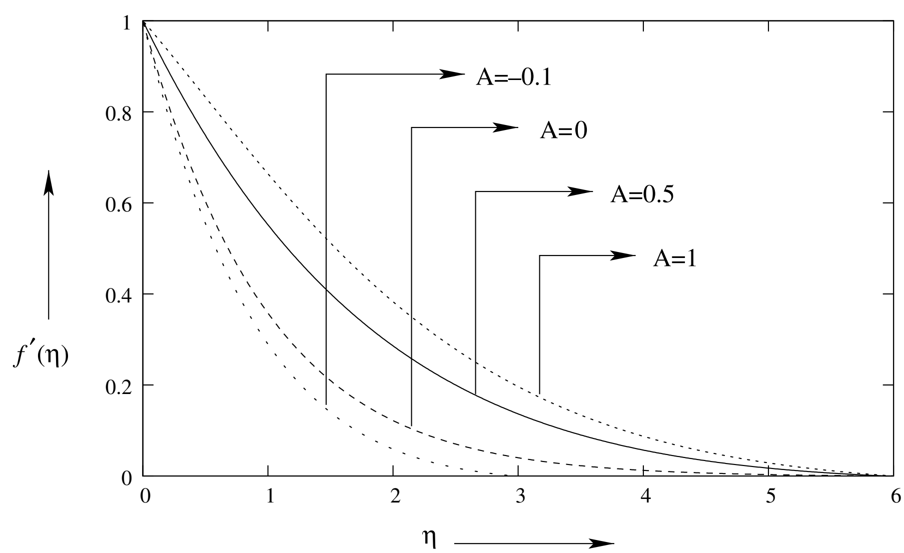

Fig. 1a. Variation of horizontal velocity $f'(\eta)$ with η for several values of A when $a = 1$, $S = 0.5$, $Pr = 0.5$ and $N = 0.1$.

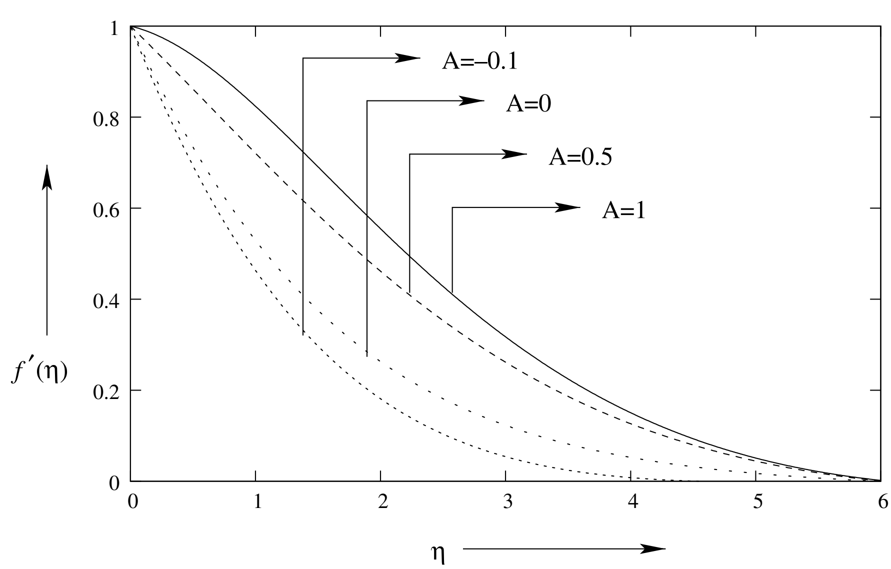

Fig. 1b. Variation of horizontal velocity $f'(\eta)$ with η for several values of A when $a = 1$, $S = -0.5$, $Pr = 0.5$ and $N = 0.1$.

3. Numerical method for solution

The above Eqs. (24) and (25) along with boundary conditions are solved by converting them to an initial value problem. We set

$$f' = z, \quad z' = p, \quad \theta' = q, \tag{28}$$

$$p' = (2z^2 - 3fp - 4A\theta + 4Apq)/(4(a + A - A\theta)),$$

$$q' = -\frac{3}{4} Pr\, fq \Big/ \left(1 + \frac{4}{3N}\right). \tag{29}$$

with the boundary conditions

$$f(0) = S, \quad f'(0) = 1, \quad \theta(0) = 1. \tag{30}$$

To integrate Eqs. (28) and (29) as an initial value problem we require a value for $p(0)$, i.e. $f''(0)$ and $q(0)$, i.e. $\theta'(0)$ but no such values are given in the boundary. The suitable guess values for $f''(0)$ and $\theta'(0)$are chosen and then integration is carried out. We compare the calculated values for f' and θ at $\eta = 6$ (say) with the given boundary condition $f'(6) = 0$ and $\theta(6) = 0$ and adjust the estimated values, $f''(0)$ and $\theta'(0)$, to give a better approximation for the solution.

We take the series of values for $f''(0)$ and $\theta'(0)$, and apply the fourth-order classical Runge–Kutta method with step-size $h = 0.01$. The above procedure is repeated until we get the results up to the desired degree of accuracy, 10^{-5}.

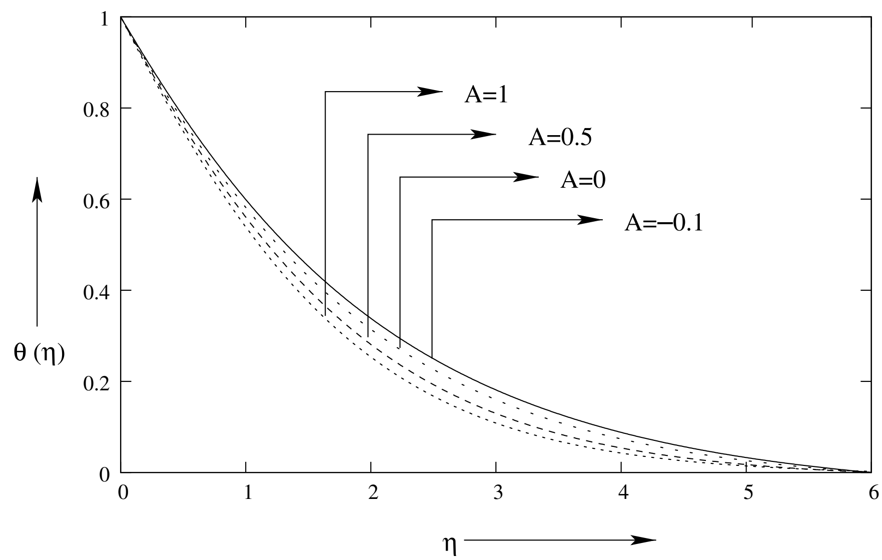

Fig. 2a. Variation of temperature $\theta(\eta)$ with η for several values of A when $a = 1$, $S = 0.5$, $Pr = 0.5$ and $N = 0.1$.

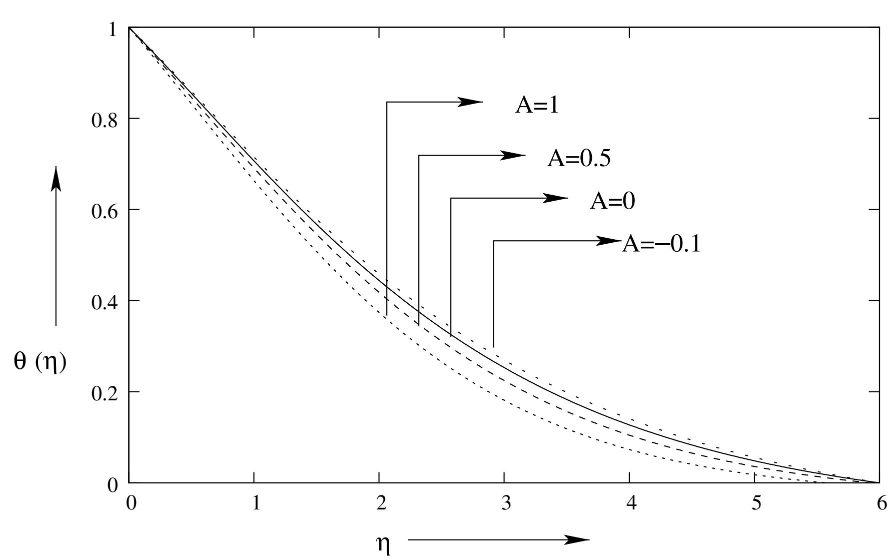

Fig. 2b. Variation of temperature $\theta(\eta)$ with η for several values of A when $a = 1$, $S = -0.5$, $Pr = 0.5$ and $N = 0.1$.

4. Results and discussion

To analyse the results, numerical computation has been carried out using the method described in the previous section for various values of the temperature-dependent viscosity parameter (A), suction/injection parameter (S), Prandtl number (Pr) and radiation parameter (N). For illustrations of the results, numerical values are plotted in the Figs. 1a–10. In all cases we take $a = 1$.

Figs. 1a and 1b exhibit the horizontal velocity profiles for several values of A ($A = -0.1, 0, 0.5, 1$) with $Pr = 0.5$ in presence of suction ($S = 0.5$) and in presence of injection ($S = -0.5$), respectively when $N = 0.1$. In each case, horizontal velocity is found to increase with the increase of the temperature-dependent fluid viscosity parameter A at a particular value of η except very near the wall as well as far away of the wall (at $\eta = 6$). This means that the velocity decreases (with the increasing value of η) at a slower rate with the increase of the parameter A at very near the wall as well as far away of the wall. This can be explained physically as the parameter A increases, the fluid viscosity decreases resulting the increment of the boundary layer thickness.

In Figs. 2a and 2b, variations of temperature field $\theta(\eta)$ with η for several values of A (with $Pr = 0.5$ and $N = 0.1$) in presence of suction ($S = 0.5$) and in presence of blowing ($S = -0.5$) are shown, respectively. It is very clear from these two figures that the temperature decreases with the increasing value of A. The increase of temperature-dependent fluid viscosity parameter (A) makes

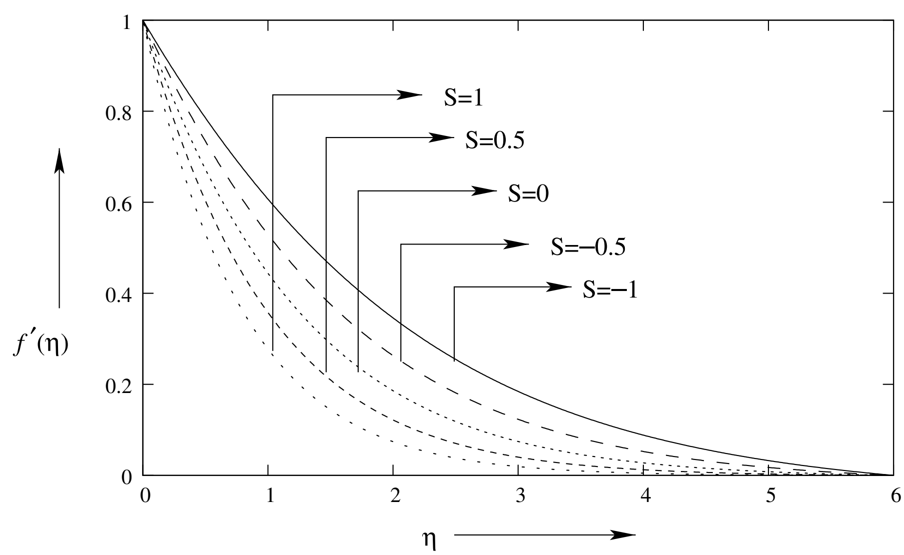

Fig. 3a. Variation of horizontal velocity $f'(\eta)$ with η for several values of S when $a = 1$, $A = 0$, $Pr = 0.5$ and $N = 0.1$.

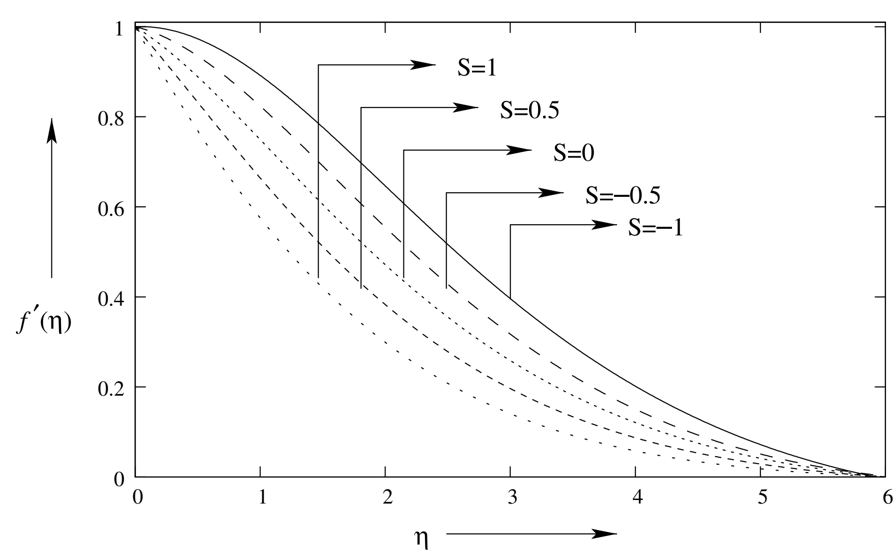

Fig. 3b. Variation of horizontal velocity $f'(\eta)$ with η for several values of S when $a = 1$, $A = 1$, $Pr = 0.5$ and $N = 0.1$.

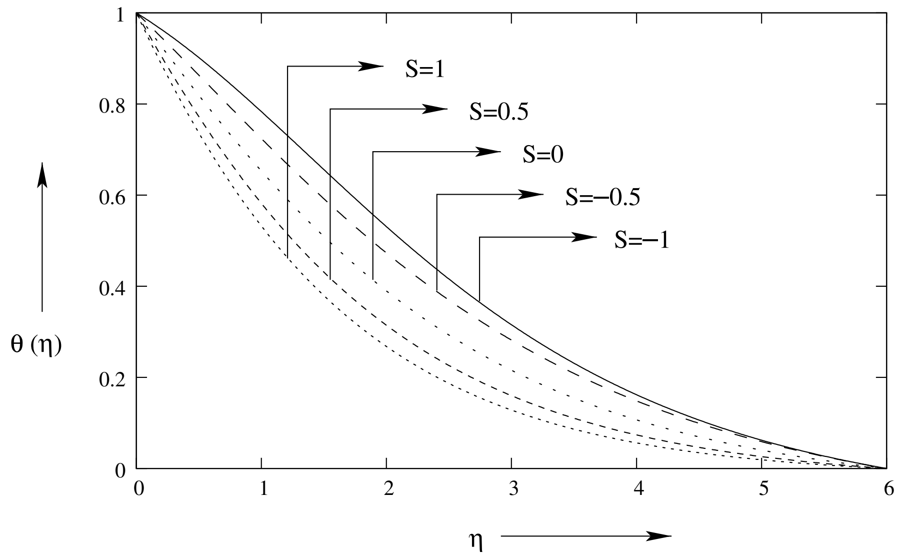

Fig. 4a. Variation of temperature $\theta(\eta)$ with η for several values of S when $a = 1$, $A = 0$, $Pr = 0.5$ and $N = 0.1$.

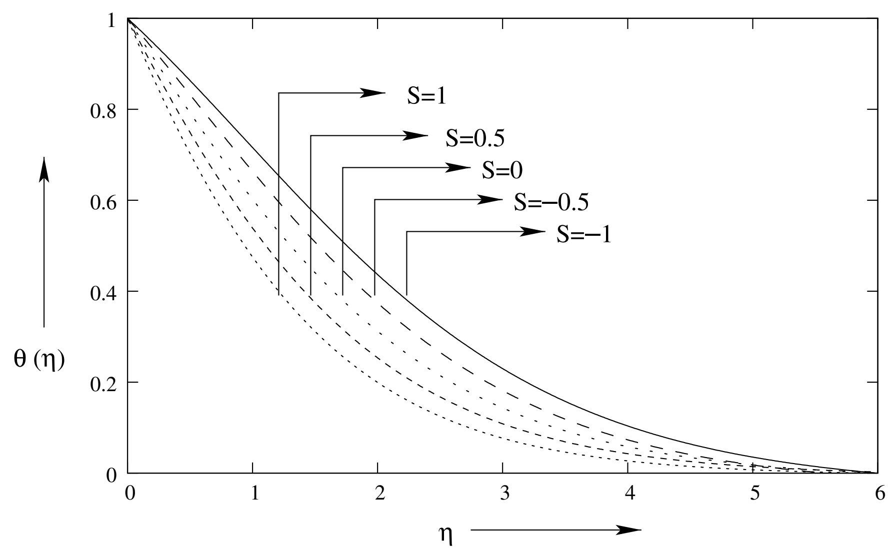

Fig. 4b. Variation of temperature $\theta(\eta)$ with η for several values of S when $a = 1$, $A = 1$, $Pr = 0.5$ and $N = 0.1$.

decrease of thermal boundary layer thickness, which results in decrease of temperature profile $\theta(\eta)$.

Decrease in $\theta(\eta)$ means a decrease in the velocity of the fluid particles. So in this case the fluid particles undergo two opposite forces: one increases the fluid velocity due to decrease in the fluid viscosity (with increasing A) and other decreases the fluid velocity due to decrease in temperature $\theta(\eta)$ (since θ decreases with increasing A). Near the surface, as the temperature θ is high so the first force dominates and far away from the surface θ is low and so the second force dominates here.

Now we concentrate in the velocity and temperature distribution for the variation of suction or injection parameter S in the absence and presence of temperature-dependent fluid viscosity parameter A.

Fig. 3a presents the effects of suction or blowing on the horizontal fluid velocity when the fluid viscosity is uniform, i.e. $A = 0$. With the increasing value of the suction ($S > 0$) [$A = 0$, $Pr = 0.5$ and $N = 0.1$], the horizontal velocity is found to decrease (Fig. 3a), i.e. suction causes to decrease the velocity of the fluid in the boundary layer region. The physical explanation for such a behaviour is as follows. In case of suction, the heated fluid is pushed towards the wall where the buoyancy forces can act to retard the fluid due to high influence of the viscosity. This effect acts to decrease the wall shear stress. But the fluid velocity increases with the increasing value of the injection parameter ($S < 0$) at a particular value of η. In this case, i.e. when stronger blowing is provided, the heated fluid is pushed farther from the wall where the buoyancy forces can act to

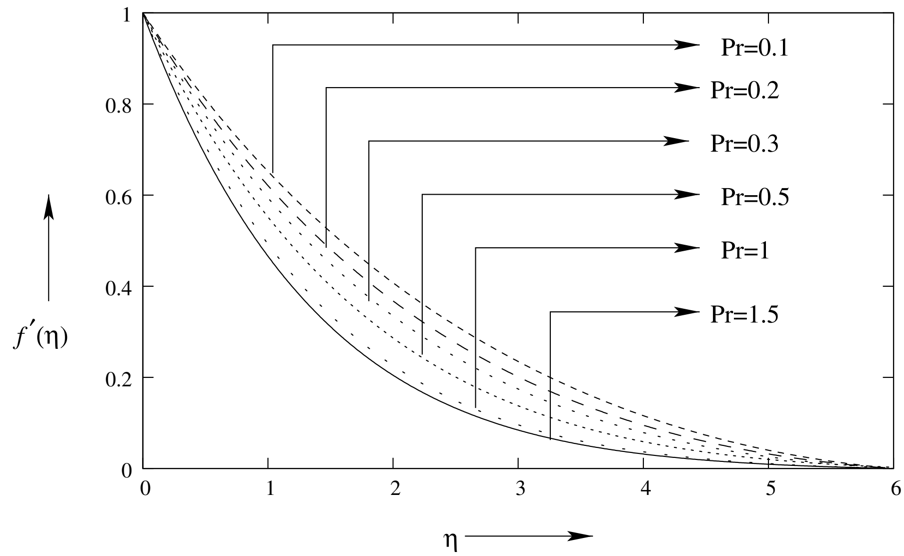

Fig. 5. Variation of horizontal velocity $f'(\eta)$ with η for several values of Pr when $a = 1$, $A = 0.5$, $S = 0.5$ and $N = 0.1$.

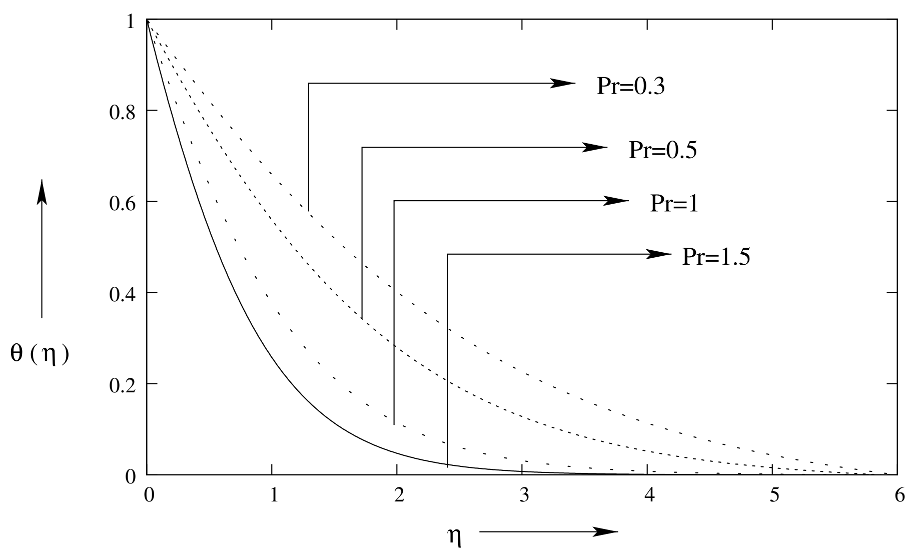

Fig. 6. Variation of temperature $\theta(\eta)$ with η for several values of Pr when $a = 1$, $A = 0.5$, $S = 0.5$ and $N = 0.1$.

accelerate the flow with less influence of viscosity. Fig. 3b represents the nature of the velocity curve with variable suction or injection parameter (S) in presence of variable fluid viscosity ($A = 1$) when $Pr = 0.5$ and $N = 0.1$. Same effects [as noticed in Fig. 3a] of suction or blowing are observed.

Fig. 4a exhibits that the temperature $\theta(\eta)$ in boundary layer also decreases with the increasing suction parameter S ($S > 0$) (the fluid is of uniform viscosity, i.e. $A = 0$) ($Pr = 0.5$ and $N = 0.1$) whereas the temperature at a particular point of the sheet increases with the increasing value of the injection parameter ($S < 0$). The thermal boundary layer thickness decreases (increases) with the suction (injection) parameter S which causes an increase (decrease) in the rate of heat transfer. The explanation for such behaviour is that the fluid is brought closer to the surface and reduces the thermal boundary layer thickness in case of suction. Fig. 4b is the graphical representation of the behaviour of the temperature field with variable suction or injection parameter S when the fluid viscosity is non-uniform ($A = 1$) [$Pr = 0.5$ and $N = 0.1$].

The effects of Prandtl number on the boundary layer flow velocity and temperature field are given in Figs. 5,6 for $A = 0.5$, $S = 0.5$ and $N = 0.1$. Fig. 5 gives the horizontal velocity profile for several values of the Prandtl number (Pr). The velocity profiles show a decrease with the increase of Prandtl number (Pr). Increase in Prandtl number means increase of fluid viscosity which causes a decrease in the flow velocity.

In Fig. 6, the variation of temperature $\theta(\eta)$ vs. η for several values of Pr is given. It is noticed that temperature decreases with the increasing value of Prandtl number

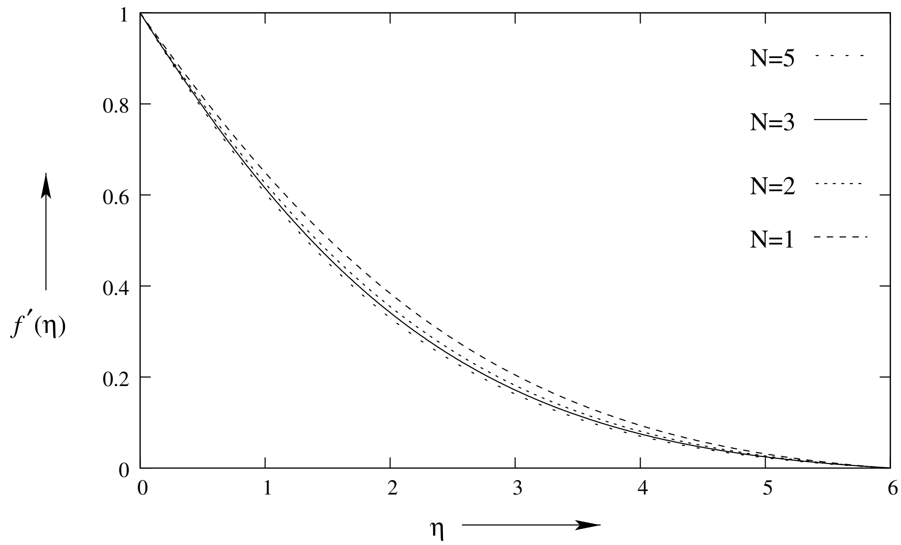

Fig. 7. Variation of horizontal velocity $f'(\eta)$ with η for several values of N when $a = 1$, $A = 0.5$, $S = 0$ and $Pr = 0.7$.

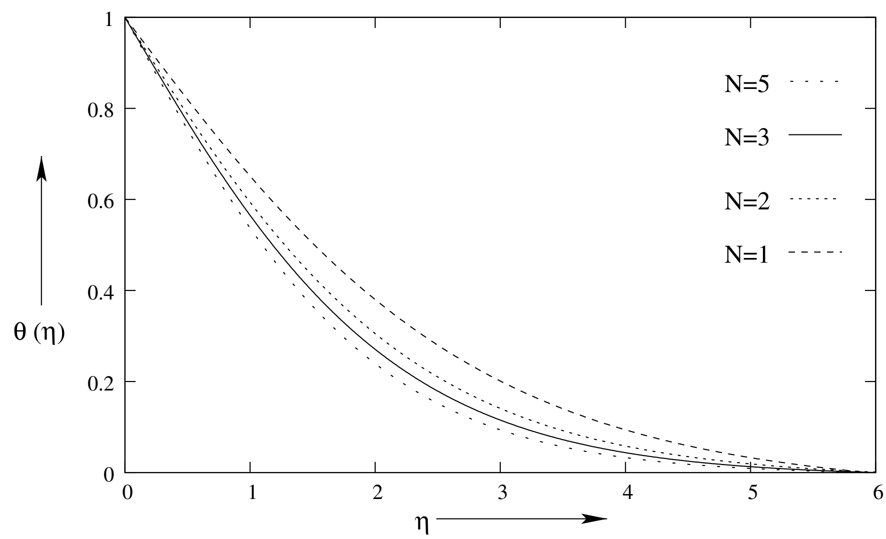

Fig. 8. Variation of temperature $\theta(\eta)$ with η for several values of N when $a = 1$, $A = 0.5$, $S = 0$ and $Pr = 0.7$.

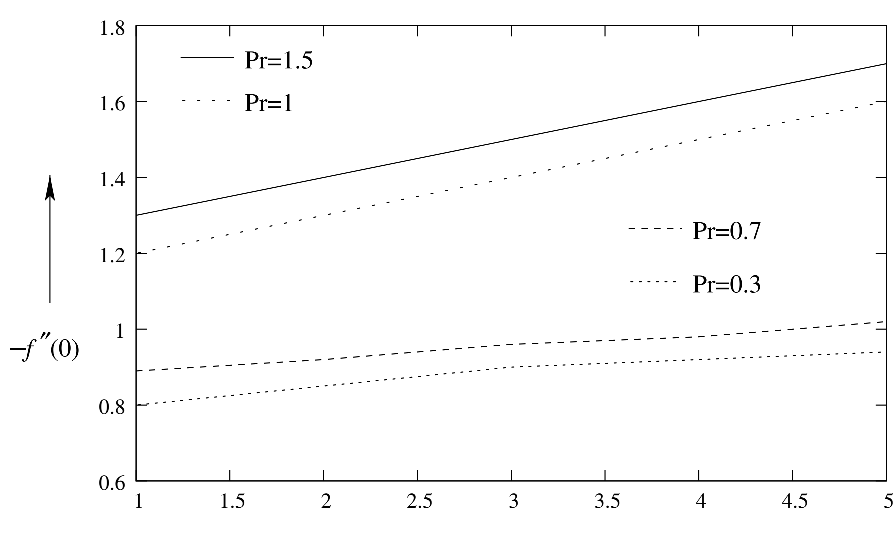

Fig. 9. Variation of skin-friction coefficient $[-f''(0)]$ with N for several values of Pr when $a = 1$, $A = 0.5$ and $S = 0$.

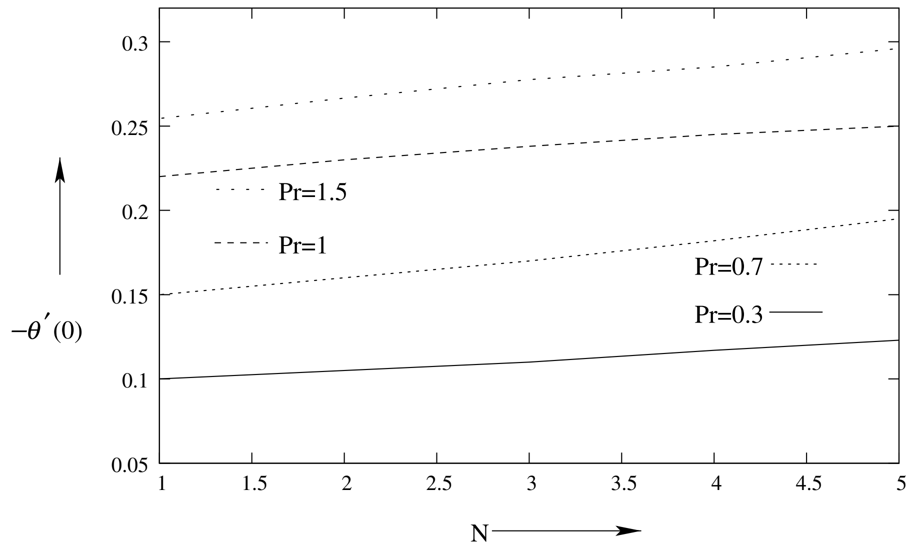

Fig. 10. Variation of rate of heat transfer $[-\theta'(0)]$ with N for several values of Pr when $a = 1$, $A = 0.5$ and $S = 0$.

because thermal boundary layer thickness decreases due to increase in Pr.

The effect of radiation parameter (N) on the velocity boundary layer characteristics is shown in Fig. 7 for $A = 0.5$, in the absence of suction ($S = 0$) [$Pr = 0.7$]. The velocity profiles show a decrease with the increase of radiation parameter (N). In Fig. 8, the variation of temperature $\theta(\eta)$ with η for various values of the radiation parameter N ($N = 1, 2, 3, 5$) with $A = 0.5$, $Pr = 0.7$ in the absence of suction is given. It is noticed from the figure that the temperature decreases with the increasing value of the radiation parameter N. The effect of radiation parameter N is to reduce the temperature significantly in the flow region. The increase in radiation parameter means the release of heat energy from the flow region and so the fluid temperature decreases as the thermal boundary layer thickness becomes thinner.

Figs. 9 and 10 depict the variations of the skin-friction coefficient $[-f''(0)]$ and rate of heat transfer $[-\theta'(0)]$ in terms of Nusselt number with radiation parameter (N) for different values of Prandtl number [$Pr = 0.3, 0.7, 1, 1.5$]. From these two figures it is noticed that both the skin-friction and rate of heat transfer increase with the increasing values of Prandtl number.

5. Conclusion

The present study gives the similarity solutions for steady free convective boundary layer flow and heat transfer over a porous stretching surface with power-law velocity distribution in presence of thermal radiation and temperature-dependent fluid viscosity. The effect of increasing temperature-dependent fluid viscosity parameter on a viscous incompressible fluid is to increase the flow velocity which in turn, causes the temperature to decrease. The results pertaining to the present study indicate that due to suction the skin-friction decreases while the rate of heat transfer increases. The temperature in the boundary layer decreases (increases) due to suction (blowing). Horizontal velocity as well as temperature decreases with the increase in Prandtl number and radiation parameter. The rate of heat transfer increases with the increasing values of Prandtl number and radiation parameter. The boundary layer edge is reached faster as Pr increases.

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